Office of Naval Research

Contract N5ORI-76 · Task Order No.1 · NR-078-011

A SHIELDED TWO-WIRE HYBRID JUNCTION

By

Edgar W. Matthews, Jr.

March 10, 1954

Technical Report No. 183

Cruft Laboratory
Harvard University
Cambridge, Massachusetts

Office of Naval Research

Contract N5-ori-76

Task Order No. 1

NR-078-011

Technical Report

on

A Shielded Two-Wire Hybrid Junction

by

Edgar W. Matthews, Jr.

March 10, 1954

The research reported in this document was made possible through support extended Cruft Laboratory, Harvard University jointly by the Navy Department (Office of Naval Research), the Signal Corps of the U. S. Army, and the U. S. Air Force, under ONR Contract N5ori-76, T. O. 1.

Technical Report No. 183

Cruft Laboratory

Harvard University

Cambridge, Massachusetts

TR183

A Shielded Two-Wire Hybrid Junction

by

Edgar W. Matthews, Jr.

Cruft Laboratory, Harvard University

Cambridge, Massachusetts

ABSTRACT

This paper describes a shielded two-wire hybrid junction and presents a theoretical analysis of its properties, with particular emphasis upon its use as the basic element of an impedance bridge. In addition, the problem of definition and measurement of impedances on this type of line, with two propagating modes, is discussed; and the line constants for the particular line configuration used are evaluated.

- - - - - -

I

SHIELDED TWO-WIRE LINE THEORY

A. Generalized Reflection Coefficients

The conventional analysis of a two-wire line usually begins with the assumption that the currents in the two wires are exactly equal in amplitude and opposite in phase at any transverse plane. In practice, however, this is seldom realized, especially when the line length is comparable to the wavelength of the applied voltage. In any case, the actual currents can be separated into two components: balanced currents, which are exactly equal and opposite; and unbalanced currents, which are equal and in the same direction in both wires. On open two-wire lines, balanced currents are true transmission-line-type currents, in that their associated electromagnetic fields cancel almost exactly at large distances from the wires, for line spacings which are a small fraction of a wavelength. On the other hand, the fields from unbalanced currents reinforce each other, and thus produce a radiation of real power. It is for this reason that two-wire transmission lines used at high frequencies are often shielded.

A shield around a two-wire line has little effect on the balanced currents, except to change their characteristic impedance; but it does eliminate the radiation of the unbalanced components by providing a return path for the currents so that they also behave like true transmission-line-type currents. This situation is usually spoken of in terms of a transmission line which has two propagating "modes," balanced and unbalanced. On a uniform, symmetrical line composed of good conductors, these two modes can exist and propagate entirely independently, just as in the case of a waveguide operating at a frequency such that more than one mode can propagate. Each mode can be considered as existing on a separate line, with its own generator and load terminations. The only possible additional factor which must be taken into consideration is the coupling between modes at the generator and the load. This can be accounted for in the separate-line model by representing both generator and load as two-terminal-pair networks, connected between the two lines. In the scattering-representation matrix for these networks, the S_{11} and S_{22} factors are in the nature of self-reflection coefficients, while the S_{12} and S_{21} factors represent coupling between modes, and can be considered as mutual reflection coefficients. Inasmuch as the two modes actually exist on the same line, this scattering matrix may be considered a type of generalized reflection-coefficient matrix, with elements Γ_{BB} and Γ_{UU} to represent the self-reflection of the balanced and unbalanced modes, respectively, and Γ_{BU} and Γ_{UB} to represent the balanced reflection from an unbalanced incident wave, and vice versa, respectively. If the component waves are normalized properly in terms of their characteristic impedances, $\Gamma_{BU} = \Gamma_{UB}$ because of reciprocity. The need for such a generalized reflection coefficient matrix with at least three independent terms is obviously brought about by the coexistence of the two modes on a single transmission line.

Coupling between the modes usually exists only because of some physical dissymmetry of the line or terminations. If this can be avoided, the two modes can be treated entirely independently. Systems with couplings between modes can rapidly become too complex for analysis; and because such couplings usually arise unintentionally, most of the following work will be done on the ideal assumption that none exists. However, after an analysis of the measurement problem concerning the component waves, a development of the generalized reflection coefficient matrices for simple tee-and pi-terminating networks will be given, to show how cross-coupling arises and may be treated.

B. Measurement of Component Waves

Following the notation of Tomiyasu [1], the potential of each of the two wires of a shielded two-wire line, with respect to the shield, may be written as:

$$V_1 = (Ae^{-\gamma_B x} + Be^{+\gamma_B x} + Ce^{+\gamma_U x} + De^{-\gamma_U x})e^{j\omega t} \qquad (1\text{-}1a)$$

$$V_2 = (-Ae^{-\gamma_B x} - Be^{+\gamma_B x} + Ce^{+\gamma_U x} + De^{-\gamma_U x})e^{j\omega t} \qquad (1\text{-}1b)$$

where A and D represent the complex amplitudes of the incident balanced and unbalanced waves, respectively, and B and C the reflected balanced and unbalanced waves, respectively. γ_B is the balanced propagation constant, and γ_U the unbalanced. The complex amplitude coefficients are formally related at the generator and load terminals by generalized reflection coefficients as described in the previous section; and when these relations are incorporated into the above set of voltage equations, the voltages are uniquely determined at each point along the line. Conversely, a given voltage distribution uniquely determines the terminating reflection coefficients; this is the usual measurement problem -- determination of the terminating impedances from the measured voltage distributions.

To date, the most accurate means of impedance measurement on long transmission lines has been by examining the voltage (or current) standing-wave distribution using a sampling technique. The usual quantities actually measured are the standing-wave ratio, ρ, and the position of the standing-wave minimum with respect to the terminal plane of the impedance being measured. On the shielded two-wire line, it is convenient to measure these same quantities on each of the lines independently. However, an examination of Eqs. (1-1) will show that there are essentially eight independent quantities, corresponding to the amplitude and phase of each of the four complex coefficients. Inasmuch as the determination of relative values only is necessary for impedance calculations, one of these coefficients may be considered arbitrary.

The measurement of the other three is then necessary for determining the three independent complex reflection coefficients of the termination; this cannot be accomplished by measurement of the two standing-wave ratios and positions alone. One alternative is to include a measurement of relative time phase, which is generally considered difficult. Another alternative is to eliminate one of the component waves during a particular measurement; this can be done to one of the incident waves only, as the reflected waves will be produced by the unknown termination itself. If the generator excites only one mode, and if the reflected wave in the second mode is completely absorbed, there will be no incident wave in the second mode. Tomiyasu [1] has shown how to eliminate the unbalanced incident wave in this manner, and gives curves for the calculation of Γ_{BB} and Γ_{UB} from the resulting measurements. In order to determine Γ_{UU} and Γ_{BU}, a second similar set of measurements has to be made, with the balanced incident wave eliminated.

C. Reflection Coefficients of Simple Terminating Networks

The possibility of representing a generalized termination of a shielded two-wire line in terms of a two-terminal-pair network connecting the two modes has just been discussed; furthermore, any two-terminal-pair network can be represented at a given frequency by a simple three-element tee or pi-network, with perhaps an ideal isolation transformer. It is the purpose of this section to develop the relationship between the elements of both a tee and a pi-terminating network and the elements of the generalized reflection coefficient matrix.

Simple tee and pi terminating networks for a shielded two-wire line are shown in Fig. 1-1; the shield constitutes the common ground connection, and the voltages are specified relative to the shield. Assumed positive directions of currents are as shown.

In the general expressions (1-1) for the voltages on the two wires, the identification of the terms A and D as incident or forward-propagating waves implies that the distance x is measured in a positive direction away from the generator. The zero reference point for x is, however, quite arbitrary; and in order to avoid unnecessary complication in the present development, let

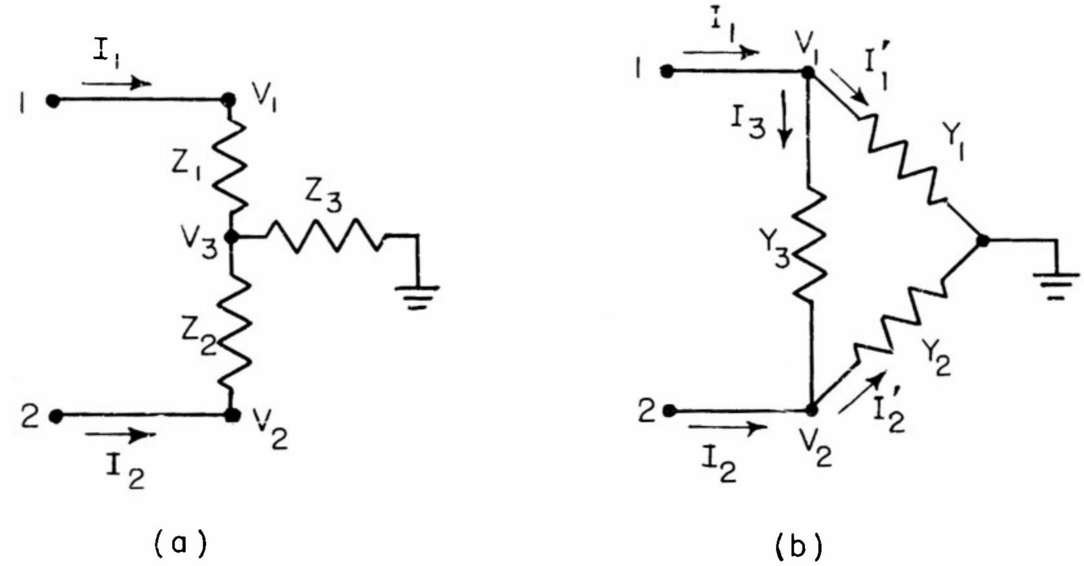

FIG. 1-1 TEE AND PI TERMINATING NETWORKS

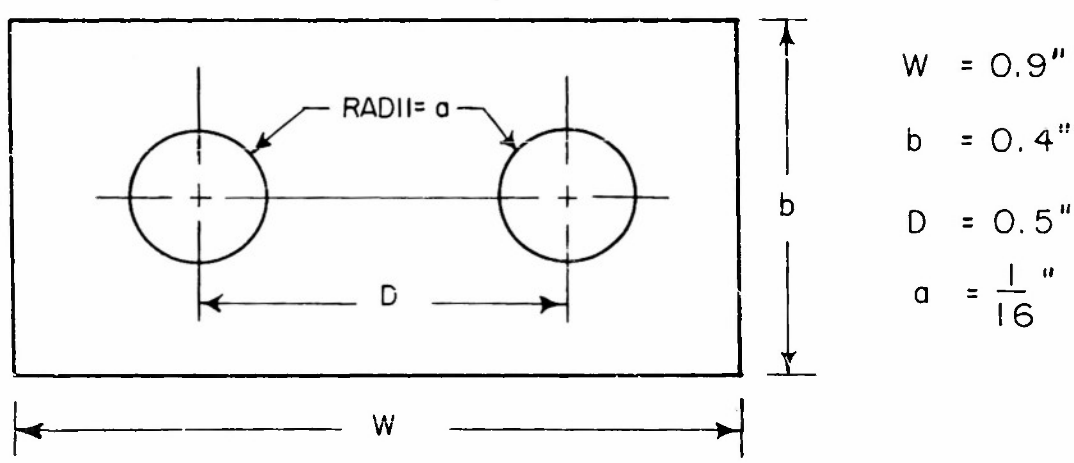

FIG. 1-2 TRANSMISSION LINE CROSS-SECTION

this zero reference point for x be defined as the terminals of the terminating network. At this point, Eqs. (1-1) reduce to:

$$V_1(0) = A + B + C + D \qquad (1\text{-}1c)$$

$$V_2(0) = - A - B + C + D \qquad (1\text{-}1d)$$

where the time dependence is implicit. The current equations can be obtained from (1-1) with the use of the relationship $Iz = I\gamma Z_c = - dV/dx$, where Z_c is the characteristic impedance of the particular mode. These become:

$$I_1 = \left[\frac{1}{Z_{cB}}(Ae^{-\gamma_B x} - Be^{\gamma_B x}) + \frac{1}{Z_{cU}}(-Ce^{\gamma_U x} + De^{-\gamma_U x})\right] e^{j\omega t}$$

$$I_2 = \left[\frac{1}{Z_{cB}}(-Ae^{-\gamma_B x} + Be^{\gamma_B x}) + \frac{1}{Z_{cU}}(-Ce^{\gamma_U x} + De^{-\gamma_U x})\right] e^{j\omega t} \qquad (1\text{-}2)$$

and at the terminals of the terminating network, these are:

$$I_1(0) = \frac{A - B}{Z_{cB}} + \frac{-C + D}{Z_{cU}}$$

$$I_2(0) = \frac{-A + B}{Z_{cB}} + \frac{-C + D}{Z_{cU}} \qquad (1\text{-}2a)$$

The characteristic impedance defined here for each mode is the ratio of the voltage of that mode on one wire, with respect to the shield, to the current of that mode in one wire. This definition is desirable in the present development for the sake of symmetry; it differs, however, from the conventional definition in the next section by a factor of two, since those definitions are made in terms of the voltage difference between wires for the balanced mode, and the total current in both wires for the unbalanced mode.

The generalized reflection coefficients may be defined in terms of the voltage equations (1-1), subject to certain conditions, as follows:

$$\Gamma_{BB} = \frac{(B)}{(A)}\bigg|_{D=0} \qquad \Gamma_{UU} = \frac{(C)}{(D)}\bigg|_{A=0}$$

$$\Gamma_{UB} = \frac{(C)}{(A)}\bigg|_{D=0} \qquad \Gamma_{BU} = \frac{(B)}{(D)}\bigg|_{A=0} \qquad (1\text{-}3)$$

1. Tee Network

The following general relations may be written for the tee network of Fig. 1-1a:

$$V_1 = I_1(Z_1 + Z_3) + I_2 Z_3 \qquad V_2 = I_1 Z_3 + I_2(Z_2 + Z_3) \qquad (1\text{-}4)$$

Expressing the voltages and currents in terms of the component waves from (1-1c), (1-1d), and (1-2a), these become:

$$A + B + C + D = \left(\frac{A-B}{Z_{cB}} + \frac{D-C}{Z_{cU}}\right)(Z_1 + Z_3) + \left(\frac{B-A}{Z_{cB}} + \frac{D-C}{Z_{cU}}\right) Z_3 \qquad (1\text{-}5)$$

$$C + D - A - B = \left(\frac{A-B}{Z_{cB}} + \frac{D-C}{Z_{cU}}\right) Z_3 + \left(\frac{B-A}{Z_{cB}} + \frac{D-C}{Z_{cU}}\right)(Z_2 + Z_3)$$

Setting D = 0, and eliminating B and C in turn gives:

$$\Gamma_{BB} = \frac{B}{A} = \frac{1}{\Delta}\left[(Z_1 - Z_{cB})(Z_2 + 2Z_3 + Z_{cU}) + (Z_2 - Z_{cB})(Z_1 + 2Z_3 + Z_{cU})\right] \qquad (1\text{-}6a)$$

$$\Gamma_{UB} = \frac{C}{A} = \frac{2 Z_{cU}(Z_1 - Z_2)}{\Delta} \qquad (1\text{-}6b)$$

where $\Delta = (Z_1 + Z_{cB})(Z_2 + 2Z_3 + Z_{cU}) + (Z_2 + Z_{cB})(Z_1 + 2Z_3 + Z_{cU})$

Setting A = 0 in (1-5) gives:

$$\Gamma_{UU} = \frac{C}{D} = \frac{1}{\Delta}\left[(Z_1 + 2Z_3 - Z_{cU})(Z_2 + Z_{cB}) + (Z_2 + 2Z_3 - Z_{cU})(Z_1 + Z_{cB})\right] \qquad (1\text{-}6c)$$

$$\Gamma_{BU} = \frac{B}{D} = \frac{2 Z_{cB}(Z_1 - Z_2)}{\Delta} \qquad (1\text{-}6d)$$

It is interesting to note that the normalized cross-coupling coefficients are equal, that is:

$$\frac{\Gamma_{UB}}{Z_{cU}} = \frac{\Gamma_{BU}}{Z_{cB}} = \frac{2(Z_1 - Z_2)}{\Delta} \qquad (1\text{-}7)$$

Furthermore, for a symmetrical tee $(Z_1 = Z_2)$, $\Gamma_{UB} = \Gamma_{BU} = 0$, and:

$$\Gamma_{BB} = \frac{Z_1 - Z_{cB}}{Z_1 + Z_{cB}} \qquad \Gamma_{UU} = \frac{Z_1 + 2Z_3 - Z_{cU}}{Z_1 + 2Z_3 + Z_{cU}} \tag{1-8}$$

which are just the usual relationships between reflection coefficients and terminating impedances for a single mode.

2. Pi-Network

The following relations may be written for the pi-network of Fig. 1-1b:

$$I'_1 = V_1 Y_1 \qquad I'_2 = V_2 Y_2 \qquad I_3 = (V_1 - V_2)\, Y_3$$

$$I_1 = I'_1 + I_3 \qquad I_2 = I'_2 - I_3 \tag{1-9}$$

Combining these to eliminate I'_1, I'_2, and I_3 gives:

$$I_1 = V_1 Y_1 + (V_1 - V_2) Y_3 \qquad I_2 = V_2 Y_2 - (V_1 - V_2) Y_3 \tag{1-10}$$

Expressing these voltages and currents in terms of component waves, from (1-1c), (1-1d), and (1-2a),

$$\frac{A - B}{Z_{cB}} + \frac{D - C}{Z_{cU}} = Y_1(A + B + C + D) + 2Y_3(A + B) \tag{1-11a}$$

$$\frac{B - A}{Z_{cB}} + \frac{D - C}{Z_{cU}} = Y_2(C + D - A - B) - 2Y_3(A + B) \tag{1-11b}$$

Setting D = 0 and A = 0, in turn, and solving as above,

$$\Gamma_{BB} = \frac{1}{\Delta'}[(Y_{cU} + Y_1)(Y_{cB} - Y_2 - 2Y_3) + (Y_{cU} + Y_2)(Y_{cB} - Y_1 - 2Y_3)] \tag{1-12a}$$

$$\Gamma_{UB} = \frac{2Y_{cB}(Y_2 - Y_1)}{\Delta'} \tag{1-12b}$$

$$\Gamma_{UU} = \frac{1}{\Delta'}[(Y_{cU} - Y_1)(Y_{cB} + Y_2 + 2Y_3) + (Y_{cU} - Y_2)(Y_{cB} + Y_1 + 2Y_3)] \tag{1-12c}$$

$$\Gamma_{BU} = \frac{2\,Y_{cU}(Y_2 - Y_1)}{\Delta'} \tag{1-12d}$$

where $\Delta' = (Y_{cU} + Y_1)(Y_{cB} + Y_2 + 2Y_3) + (Y_{cU} + Y_2)(Y_{cB} + Y_1 + 2Y_3)$.

Again,

$$\frac{\Gamma_{UB}}{Y_{cB}} = \frac{\Gamma_{BU}}{Y_{cU}} = \frac{2(Y_2 - Y_1)}{\Delta'} \tag{1-13}$$

or,

$$\frac{\Gamma_{UB}}{Z_{cU}} = \frac{\Gamma_{BU}}{Z_{cB}} = \frac{2(Y_2 - Y_1)}{Z_{cU}Z_{cB}\,\Delta'} \tag{1-13a}$$

And for a symmetrical pi, $(Y_1 = Y_2)$, $\Gamma_{UB} = \Gamma_{BU} = 0$, and

$$\Gamma_{BB} = \frac{Y_{cB} - Y_1 - 2Y_3}{Y_{cB} + Y_1 + 2Y_3} \qquad \Gamma_{UU} = \frac{Y_{cU} - Y_1}{Y_{cU} + Y_1} \tag{1-14}$$

D. Transmission-Line Constants

In the conventional development of transmission-line equations, a number of quantities are usually introduced which have some physical significance and at the same time specify the electrical properties of the line. These quantities are Z_o, the characteristic impedance, and $\gamma = \alpha + j\beta$ the complex propagation constant, composed of α, the attenuation constant, and β, the phase constant. In order to measure any actual impedances on a transmission line, it is necessary to know these constants for the line. A set of such constants may be defined for each mode which propagates on the line. These constants will now be determined for the particular type of line used for the hybrid junction as shown in Fig. 1-2.

1. Characteristic Impedance

The characteristic impedance is defined as the complex ratio of voltage to current at any point on a uniform transmission line of infinite length. Alternatively, it is that impedance which will terminate a transmission line of finite length without reflections. For a low-loss line, Z_o is predominantly real, and will be considered as such here.

The theoretical determination of Z_o is simplified by making use of the fact that for any transverse electromagnetic mode on a perfectly conducting transmission line, the characteristic impedance is directly related to the capacitance per unit length by the formula:

$$Z_o = \frac{j\omega\mu\epsilon}{\gamma C} = \frac{j\omega\mu\epsilon}{j\beta C} = \frac{1}{v_o C} \simeq \frac{10^{-8}}{3C} \qquad (1\text{-}15)$$

where Z_o is given in ohms if C is in farads per meter [2].

Frankel [3] gives the characteristic impedance of a two-wire line in a rectangular shield for the balancedmode only. His method of calculation is based upon a consideration of the electrostatic potential for a set of four line charges λ, of alternate polarity, located in cross section on the real axis of the complex w-plane at $\pm\alpha$ and $\pm\beta$. This potential is given by:

$$V = -2\lambda \ln \frac{|w-\alpha|\,|w+\beta|}{|w+\alpha|\,|w-\beta|} \qquad (1\text{-}16)$$

If the conformal transformation $w = e^{kz}$ is made, the four-line charges transform into an infinite set in the z-plane, corresponding to the images of a two-wire line between parallel conducting planes.

In order to determine the capacitance per unit length for this configuration, it is necessary to evaluate the potential on the surface of the wires, which are assumed to be perfect conductors of circular cross section. It is then necessary to require the radius of the wires to be small compared to the distance between them and to the distance to any side of the shield, for two reasons: to permit use of the fact that small circles in the z-plane transform into circles in the w-plane; and second, to permit the distance from one wire to any point on the surface of the other to be replaced by the distance between centers. This approximation is equivalent to the assumption that the angular charge distribution around the wires is uniform. With these assumptions, the potential between wires is:

$$V_2 - V_1 = 4\lambda \ln \left(\frac{2b}{\pi a} \tanh \frac{\pi D}{2b}\right) \qquad (1\text{-}17)$$

where λ = charge on each wire per unit length

b = distance between parallel conducting planes

D = distance between wires

a = radius of wires

The presence of the vertical walls of the shield can be accounted for by the method of images, using an infinite series of images extending to $\pm\infty$. The contribution of each pair of images to the potential difference between the original wires is found, and a summation taken. Then using (1-15) and the relation $\lambda = C(V_2 - V_1)$, Z_o for the balanced mode becomes

$$Z_{oB} = 120 \ln\left(\frac{2b}{\pi a}\tanh\frac{\pi D}{2b}\right) - 120\sum_{m=1}^{\infty}\ln\frac{1+\left[\dfrac{\sinh\frac{\pi D}{2b}}{\cosh\frac{m\pi w}{2b}}\right]^2}{1-\left[\dfrac{\sinh\frac{\pi D}{2b}}{\sinh\frac{m\pi w}{2b}}\right]^2} \tag{1-18}$$

where w = distance between vertical walls, shown in Fig. 1-2.

The same procedure can be used to obtain Z_o for the unbalanced mode, the only difference being in the sign of the charges on the wires and images. Equation (1-16) for the potential at any point in the w-plane becomes:

$$V = -2\lambda \ln\frac{|w-\alpha|\,|w-\beta|}{|w+\alpha|\,|w+\beta|} \tag{1-19}$$

and the potential of both wires above ground (the shield) is then:

$$V_1 = V_2 = 2\lambda \ln\left(\frac{2b}{\pi a}\coth\frac{\pi D}{2b}\right) \tag{1-20}$$

for the configuration between two parallel conducting planes. Including the effect of the images for the vertical walls gives Z_o for the unbalanced mode as:

$$Z_{oU} = 30\ln\left(\frac{2b}{\pi a}\coth\frac{\pi D}{2b}\right) + 30\sum_{m=1}^{\infty}(-1)^m\ln\frac{1+\left[\dfrac{\cosh\frac{\pi D}{2b}}{\sinh\frac{m\pi w}{2b}}\right]^2}{1-\left[\dfrac{\cosh\frac{\pi D}{2b}}{\cosh\frac{m\pi w}{2b}}\right]^2} \tag{1-21}$$

The transmission line used in this research consists of two 1/8-inch diameter brass wires spaced 1/2 inch apart, enclosed in a standard 3 cm waveguide. The dimensions as shown in Fig. 1-2 are thus: w = 0.900''; b = 0.400''; D = 0.500''; and a = 0.0625''. With these dimensions, the formulas for Z_o converge quickly to yield the values:

$$Z_{oB} = 153.852 \text{ ohms} \qquad Z_{oU} = 40.629 \text{ ohms}$$

As a check chiefly on the validity of the assumptions involved in the foregoing development as applied to the particular case of interest, a numerical method is available for the calculation of an approximate value of Z_o corresponding to a particular set of dimensions. This is known as the Relaxation Method, as described by Southwell [4]; it is particularly simple in this case because of the rectangular boundaries, but can be applied to more complicated boundaries with little difficulty. The method is used here to obtain a solution to Laplace's equation in two dimensions within a bounded region, with the value of the electrostatic potential given on the boundaries. The method consists of finding an approximate value for the potential at any point in the region by properly averaging the potentials at surrounding points. A plot is first made of the cross section of the transmission line, and a rectangular grid of points separated by a distance h is superimposed. The potential at each point is judiciously approximated, and then successively improved by going over the entire net, replacing the potential at each point by the average of the surrounding points, until the change in successive values is within the accuracy desired. For points near the circular boundaries of the wires (whose boundaries do not coincide with points of the rectangular net), a modified form is used for averaging, weighting each of the surrounding points according to its distance from the point being improved.

For the line with the dimensions given above, the first approximation was made with a point spacing h = 0.10 inch. Values of potential thus obtained were used for the first approximation in a second net with h = 0.05 inch, and similarly for a third net with h = 0.025 inch. For each net, the normal derivative of the potential for each net point on the boundary was calculated, and the total charge on the boundary determined by Simpson's rule for integration, from which the capacitance per unit length was calculated. The final net (h = 0.025 inch) for both balanced and unbalanced modes is reproduced in Figs. 1-3 and 1-4, respectively, showing a number of equipotential lines. Only one quarter of the cross section is shown because of symmetry. The numbers below each of the net points represent potentials, while those outside the boundaries represent values of the normal derivative.

The accuracy of the final net can be further improved by extrapolation, making use of the fact that the errors involved vary as h^2. Application of such a relationship to the results gives the following values for Z_o:

	Z_{oB}(ohms)	Z_{oU}(ohms)
First net	183.44	45.767
Second net	161.23	41.738
Third net	155.98	40.895
Extrapolated value	154.27	40.626
Theoretical value	153.85	40.629

By comparison with the theoretical values, it can reasonably be said that extrapolated values are within 0.1 per cent of the true value.

2. Attenuation Constant

The attenuation constant α is a measure of the power loss per unit length along a transmission line. For a shielded line, assuming the thickness of the shield to be large compared to the skin depth, this loss is entirely due to the resistance of the conductors. An approximate value of this power loss can be obtained as the product of the square of the total current flowing and the ohmic resistance of the line per unit length; an exact solution must take into account the non-uniform current distribution around the cross section of the transmission line. A matched line is, of course, implied, so that there are no additional losses due to the presence of standing waves.

At 750 megacycles the skin depth in brass is $5.3 \cdot 10^{-6}$ meters, or .00021 inch, which is certainly small compared to the .050-inch wall thickness, and to the .0625-inch wire radius; so the surface impedance as defined by King [5] is an adequate measure of the resistance per unit length.

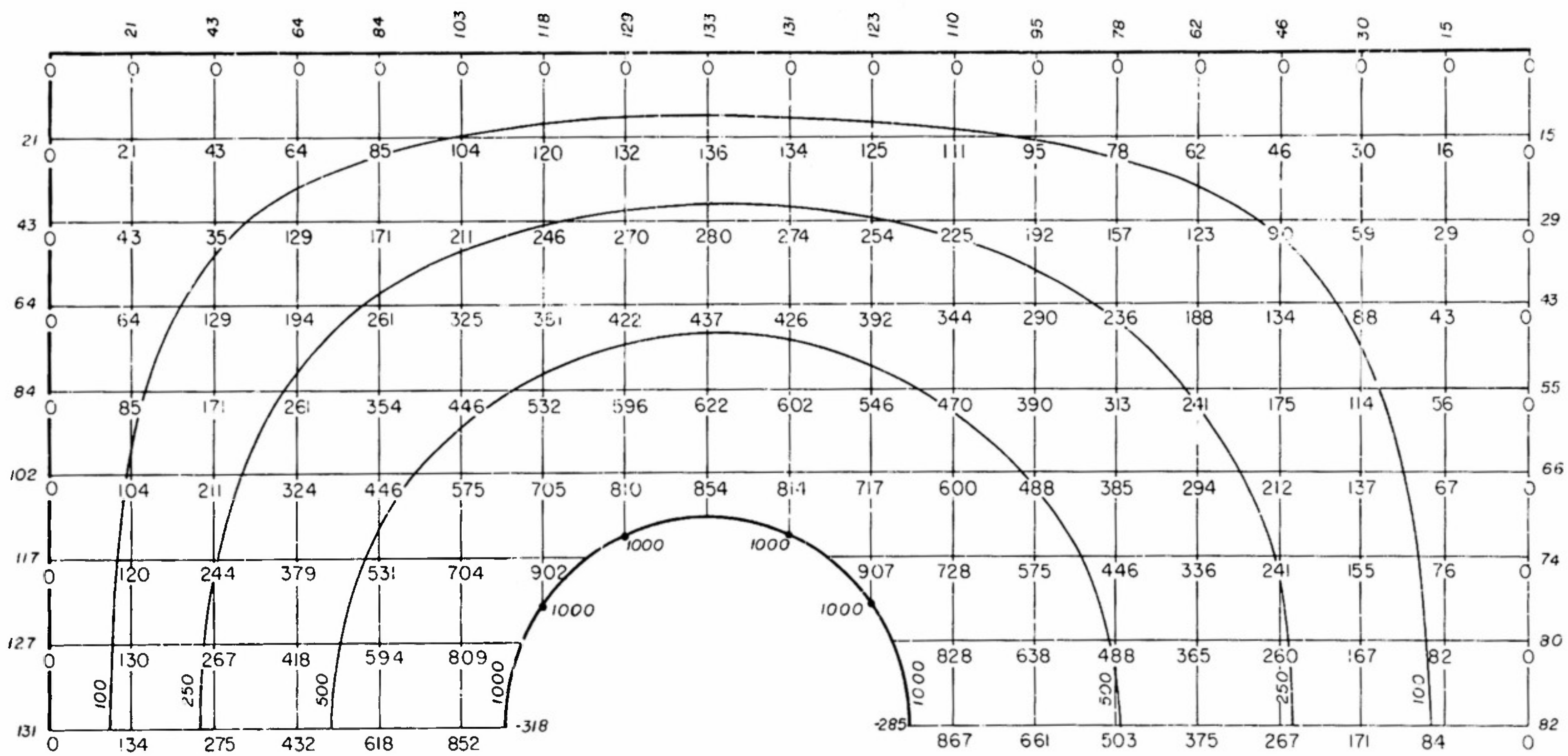

FIG. I-3 BALANCED MODE POTENTIAL DISTRIBUTION

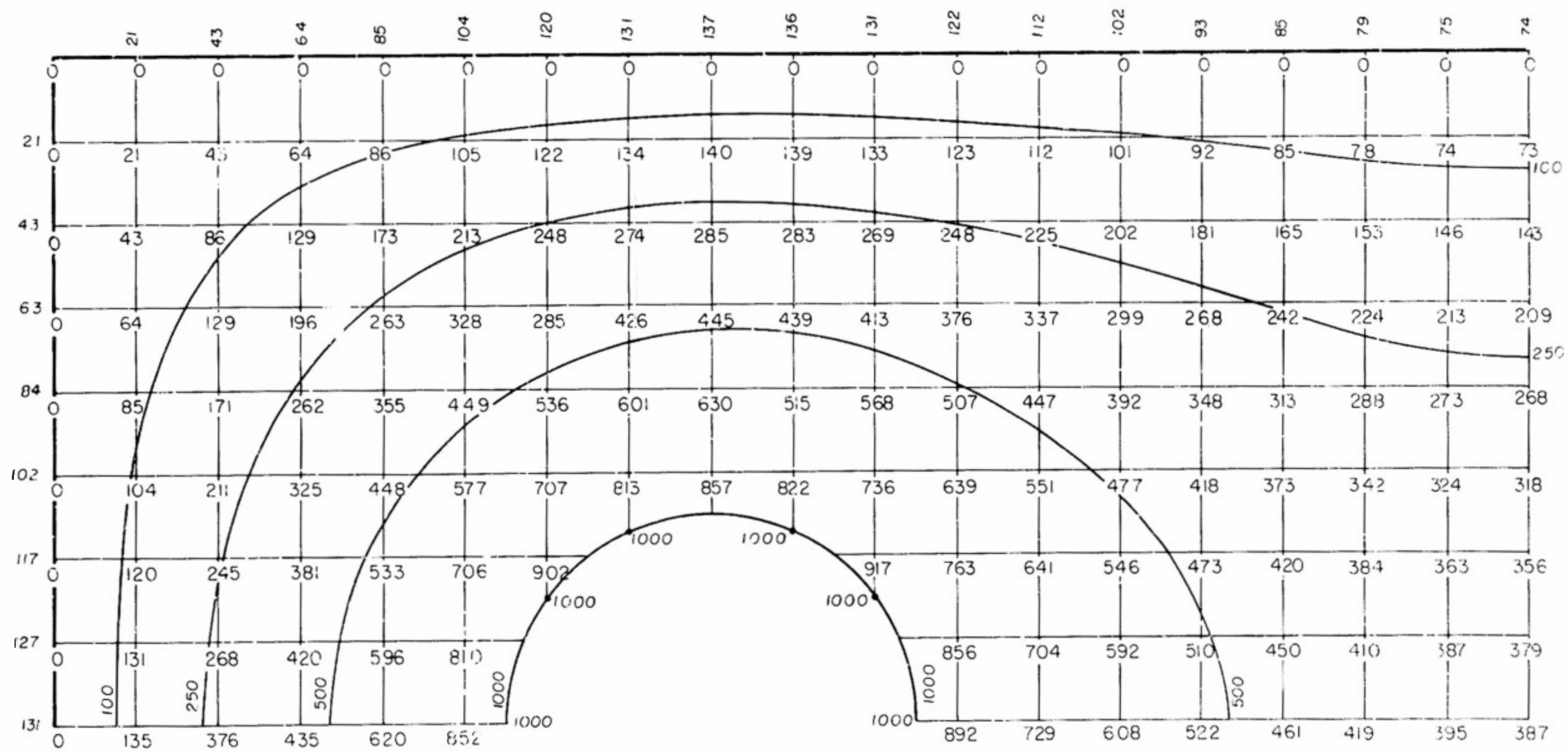

FIG I-4 UNBALANCED MODE POTENTIAL DISTRIBUTION

This is given by:

$$R^s = X^s = \sqrt{\frac{\omega}{2\nu\sigma}} \doteq \sqrt{\frac{2\pi \cdot 750 \cdot 10^6 \cdot 1.256 \cdot 10^{-6}}{2 \cdot 1.2 \cdot 10^7}} = .0157 \text{ ohms} \tag{1-22}$$

The resistance of each element of the line, R_i, is then inversely proportional to its cross-sectional perimeter, or 1.57 ohms/meter for each wire, and 0.238 ohms/meter for the shield.

The current at each point on the surface of the wires can be determined from the potential plots shown in Figs. 1-3 and 1-4 calculated by the relaxation method, using the formulas for the surface-current density, K,

$$K = n \times H \qquad E = -\frac{\partial \phi}{\partial n} \qquad E = \zeta H \qquad \zeta^2 = \frac{1}{\nu \epsilon} \tag{1-23}$$

The normal derivatives of the potential as given in the above figures, which may be called $\frac{\partial \phi'}{\partial n}$, are related to the actual $\frac{\partial \phi}{\partial n}$ by a factor $1/h$, where h is the net spacing. Consequently,

$$K = -\frac{1}{h\zeta} \frac{\partial \phi'}{\partial n} \tag{1-24}$$

The total mean-square current on each conductor is then:

$$I_i^2 = \frac{1}{h^2\zeta^2} \oint_{s_i} \frac{\partial \phi'^2}{\partial n} ds \tag{1-25}$$

where the line integral is taken completely around the surface of the conductor. A numerical method of integration is used on the results of the relaxation calculation of the normal potential derivatives; and a normalizing factor N is incorporated to correct for differences between the total current on each conductor as found by the relaxation method, and the total current which would flow on a matched line with the same applied voltage. N will be the square of the ratio of these two currents. The attenuation constant is then given by:

$$\alpha = -\frac{1}{2P} \frac{dP}{dz} \text{ nepers per unit length} \tag{1-26}$$

where P = total power delivered to matched load = V^2/Z_o

dP/dz = power loss per unit length = $\sum I_i^2 R_i$

Table 1-1 gives the results of the attenuation calculations.

3. Phase Constant β

The phase constant β is a measure of the phase change per unit length on the line. For a transverse electromagnetic wave, it has the same value as in free space, or in a uniform infinite dielectric medium, as given by:

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{v_o} = \omega\sqrt{\mu\epsilon} = \frac{\omega\sqrt{\epsilon_r}}{3\cdot 10^8} \quad \text{radians per meter} \qquad (1\text{-}27)$$

TABLE 1-1

ATTENUATION CONSTANTS OF SHIELDED TWO-WIRE LINE

	Balanced	Unbalanced
Total I^2 on each wire	173.33	157.28 amp.^{2}
Total I on each wire	12.841	12.188 amp.
Total I for matched line, per wire	12.964	12.307 amp.
Normalizing factor N	1.0192	1.0196
Normalized I^2 on each wire	176.66	160.36 amp.^{2}
Total I^2 on shield	587.60	705.66 amp.^{2}
Total I on shield	25.630	24.439 amp.
Normalizing factor N	1.0234	1.0143
Normalized I^2 on shield	601.35	715.75 amp.^{2}
Power loss in wires per meter	556.16	504.84 watts
Power loss in shield per meter	142.94	170.13 watts
Total power loss per meter	699.10	674.97 watts
Power delivered to matched load	25,929	24,615 watts
Attenuation constant α	.01348	.01371 neper/m.
α for uniform angular current distribution	.01234	.01261 "
α for copper wires	.00764	.00812 "

II

SHIELDED TWO-WIRE HYBRID JUNCTION

A. General Properties

The shielded two-wire hybrid junction, which is the basic element of the two-wire impedance bridge under investigation, is illustrated schematically in Fig. 2-1. It consists essentially of a main transmission line A-B with a pair of series connections C and D and a shunt connection E in the same transverse plane of the main line A-B. The two series connections, one in each wire of the main line, are necessary to maintain the symmetry of the junction for balanced currents on the two wires; whereas a single series connection suffices in a waveguide hybrid junction, since only a single mode will propagate. The shunt connection can be made at the same point on the main line as the series connections by making the shunt connection to the center of a short-circuited quarter-wave stub across each of the series connections.

A hybrid junction, when properly terminated, has the peculiar properties of a class of four-terminal-pair networks known as bi-conjugate networks [6], namely, that of effective isolation or decoupling between both pairs of opposite terminals. Probably the earliest example of such networks was the hybrid coil used in telephone repeater stations. The first UHF model took the form of the hybrid ring, or "rat-race" [7], consisting of a closed transmission-line ring or loop with four connections spaced around the circumference such that signals launched in opposite directions around the ring from one input terminal arrive at the opposite terminal in the proper phase to cancel each other. This cancellation is usually accomplished by making the difference in the two path lengths equal to a half-wavelength, so that the entire structure is very frequency-sensitive. Variations of this basic design are possible by insertion or removal of pairs of half-wavelength sections of lines, or by exchange of shunt connections to the ring for series connections, together with a quarter-wavelength change in position. Particular combinations of such changes can produce variations which are physically symmetrical, and so not essentially frequency-sensitive. Coaxial, waveguide, and two-wire line models of the hybrid ring have been built successfully. Other examples of bi-conjugate networks are the directional coupler and the waveguide and coaxial [8] hybrid junction, or "Magic-Tee."

The two-wire hybrid junction described above has the desirable feature of being essentially frequency-insensitive, since its properties are based chiefly on physical symmetry alone. The only frequency-sensitive elements are the quarter-wave stubs which permit series and shunt connections at the same point (without resorting to such mismatched structures as current loops). However, for frequencies from 0.5 to 1.5 times the design frequency, these stubs effectively shunt a fairly high reactance across the series arms, which can be matched out if necessary. The actual construction of the experimental model is such that the position of the short-circuiting bar terminating these stubs is adjustable (by means of a threaded rod inserted from either end of the shunt arm), so that the length of the stubs can be made equal to a quarter-wavelength from 200 to 1000 megacycles.

The bi-conjugate properties of a hybrid junction make it useful as an impedance-measuring device. Power fed into either the series or the shunt arm will not appear at the other if identical loads are placed on the two symmetrical arms. If the two loads are not identical, unequal reflections will couple part of the input power into the fourth arm. In particular, if one of the loads is perfectly matched, the output power from the fourth arm will be directly proportional to the reflection coefficient of the other load, which is one method of using the junction for an impedance bridge. Alternatively, if one of the two loads is an adjustable calibrated standard, it can be adjusted for zero output from the fourth arm, at which point it must be identical to the other load, whose value can then be determined from the calibration. This latter scheme was used for the shielded two-wire impedance bridge. Because of the unique property of this type of line concerning its ability to support two modes of propagation, the performance of the shielded two-wire hybrid junction will be investigated in more detail.

B. Theoretical Analysis

The completely rigorous analysis of the shielded two-wire hybrid junction would require the solution of Maxwell's equations subject to the appropriate boundary conditions. Certain simplifications are possible, how-

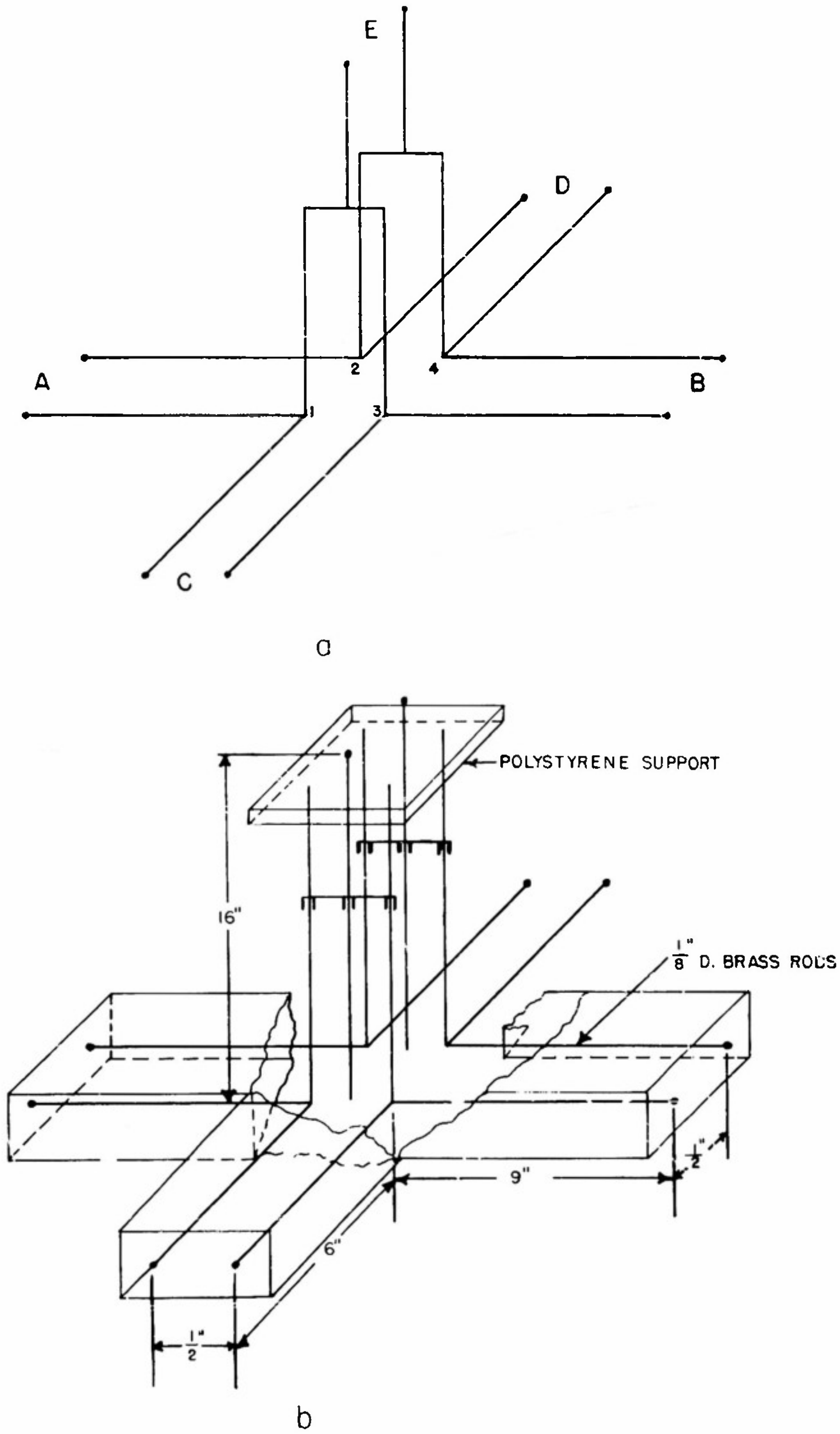

FIG 2-1 SHIELDED TWO-WIRE HYBRID JUNCTION

ever. First, it can be shown from the relative dimensions that two, and only two, modes of propagation are possible on the shielded two-wire line within the desired frequency range (200 to 1000 mc.). Furthermore, assuming perfect conductors, a unique voltage and current for each mode can be defined at each point along a long, uniform section of this line. This is not true near any discontinuities because of the excitation of other modes, which are attenuated as they travel away from the discontinuity, or alternatively, because of coupling to adjacent parts of the circuit which are not continuations of the uniform transmission line. The usual method of solution in the vicinity of a discontinuity is to solve Maxwell's equations by a variational or integral-equation method, and match this solution to a uniform-transmission-line type of solution at a point some distance away from the discontinuity. This distance in the case of a two-wire line is of the order of ten times the line spacing; and if this distance is small compared with a wavelength, the discontinuity may be replaced by an equivalent lumped circuit. Experimental verification for the validity of this replacement has been obtained for the case of a simple shunt tee in the two-wire line described above, at 750 megacycles. Inasmuch as the hybrid junction is merely a combination of series and shunt tees, it seems reasonable to insert such a lumped equivalent circuit in each arm of the junction and analyze the junction completely in terms of transmission line circuits.

Best utilization of the symmetry properties of the junction for use as an impedance bridge can be made by connecting the two impedances to be compared on the symmetrical arms of the junction. Furthermore, because there are two series arms, null detectors will be placed on both of these; and the single shunt arm will be used for the input. Thus, referring to Fig. 2-1, the input will be at arm E, the two impedances for comparison on arms A and B, and the detectors on arms C and D. The four junction points will be numbered as shown on the figure; and the positive direction of current flow will be taken into the junction from arm E, and away from the junction for the others. The voltage at each junction point will be specified with respect to the shield, or ground, terminal. The following designations will be made:

V_n = voltage at the n'th junction point,

I_{mB} = balanced current component at the m'th terminal pair,

I_{mU} = unbalanced current component at the m'th terminal pair,

V_{mB} = balanced voltage component at the m'th terminal pair, with respect to ground,

V_{mU} = unbalanced voltage component at the m'th terminal pair, with respect to ground,

I_{mn} = current flowing into the m'th terminal pair from the n'th junction point.

By the definition of balanced and unbalanced components of voltage and current, and from the orientation of the terminals as shown in Fig. 2-1, the following definitions may be written:

$$\begin{aligned} V_{AB} &= \tfrac{1}{2}(V_1 - V_2) & V_{AU} &= \tfrac{1}{2}(V_1 + V_2) \\ V_{BB} &= \tfrac{1}{2}(V_3 - V_4) & V_{BU} &= \tfrac{1}{2}(V_3 + V_4) \\ V_{CB} &= \tfrac{1}{2}(V_1 - V_3) & V_{CU} &= \tfrac{1}{2}(V_1 + V_3) \\ V_{DB} &= \tfrac{1}{2}(V_2 - V_4) & V_{DU} &= \tfrac{1}{2}(V_2 + V_4) \end{aligned} \tag{2-1}$$

$$\begin{aligned} I_{AB} &= \tfrac{1}{2}(I_{A1} - I_{A2}) & I_{AU} &= \tfrac{1}{2}(I_{A1} + I_{A2}) \\ I_{BB} &= \tfrac{1}{2}(I_{B3} - I_{B4}) & I_{BU} &= \tfrac{1}{2}(I_{B3} + I_{B4}) \\ I_{CB} &= \tfrac{1}{2}(I_{C1} - I_{C3}) & I_{CU} &= \tfrac{1}{2}(I_{C1} + I_{C3}) \\ I_{DB} &= \tfrac{1}{2}(I_{D2} - I_{D4}) & I_{DU} &= \tfrac{1}{2}(I_{D2} + I_{D4}) \end{aligned} \tag{2-2}$$

$$I_{EB} = \tfrac{1}{2}(I_{E1} + I_{E3} - I_{E2} - I_{E4})$$

$$I_{EU} = \tfrac{1}{2}(I_{E1} + I_{E2} + I_{E3} + I_{E4})$$

From the requirements for conservation of current at each junction point, the following relations may be written:

$$I_{E1} = I_{A1} + I_{C1} \qquad I_{E2} = I_{A2} + I_{D2}$$

$$I_{E3} = I_{B3} + I_{C3} \qquad I_{E4} = I_{B4} + I_{D4} \tag{2-3}$$

Because of the quarter-wave stub in each E-terminal line,

$$I_{E1} = I_{E3} \qquad I_{E2} = I_{E4}$$

$$\therefore \quad I_{EB} = I_{E1} - I_{E2} \qquad I_{EU} = I_{E1} + I_{E2} \tag{2-4}$$

It is now desired to solve the above equations for the detector voltages and/or currents (V_C, V_D, I_C, and I_D) in terms of the driving currents I_{EB} and I_{EU} and the impedances connected to the various arms. In general, these impedances will have to be expressed in terms of impedance, admittance, or reflection-coefficient matrices, relating both balanced and unbalanced components of voltage and current, as outlined in Chapter I. This generality will greatly complicate the solution; and inasmuch as impedances of interest are usually symmetrical, this solution will be restricted to the case of no coupling between modes in the terminating impedances -- i.e., $\Gamma_{BU} = \Gamma_{UB} = 0$. If these impedances are specified with respect to a reference plane at the actual junction points, and include the effects of the equivalent lumped circuit of the junction as described above, the following relations may be written:

$$I_{AB} = Y_{AB} V_{AB} \qquad I_{AU} = Y_{AU} V_{AU}$$

$$I_{BB} = Y_{BB} V_{BB} \qquad I_{BU} = Y_{BU} V_{BU}\text{, etc.} \tag{2-5}$$

The solution can be carried out in two steps, one with $I_{EB} = 0$ and the other with $I_{EU} = 0$, and the two combined according to the law of superposition.

1. For the case with $I_{EB} = 0$, from (2-4),

$$I_{E1} = I_{E2} = I_{E3} = I_{E4} = \frac{I_{EU}}{2} \tag{2-6}$$

and from (2-3),

$$I_{A1} + I_{C1} = I_{A2} + I_{D2} = I_{B3} + I_{C3} = I_{B4} + I_{D4} = \frac{1}{2} I_{EU} \tag{2-7}$$

I_A and I_B can then be eliminated from (2-2) to give:

$$\begin{aligned} I_{AB} &= Y_{AB} V_{AB} = \tfrac{1}{2}(I_{A1} - I_{A2}) = \tfrac{1}{2}(I_{D2} - I_{C1}) \\ I_{AU} &= Y_{AU} V_{AU} = \tfrac{1}{2}(I_{A1} + I_{A2}) = \tfrac{1}{2}(I_{EU} - I_{C1} - I_{D2}) \\ I_{BB} &= Y_{BB} V_{BB} = \tfrac{1}{2}(I_{B3} - I_{B4}) = \tfrac{1}{2}(I_{D4} - I_{C3}) \\ I_{BU} &= Y_{BU} V_{BU} = \tfrac{1}{2}(I_{B3} + I_{B4}) = \tfrac{1}{2}(I_{EU} - I_{C3} - I_{D4}) \end{aligned} \tag{2-8}$$

But from (2-1),

$$\begin{aligned} V_1 &= V_{CB} + V_{CU} \qquad & V_2 &= V_{DB} + V_{DU} \\ V_3 &= V_{CU} - V_{CB} \qquad & V_4 &= V_{DU} - V_{DB} \end{aligned} \tag{2-9}$$

And from (2-2),

$$\begin{aligned} I_{C1} &= I_{CB} + I_{CU} \qquad & I_{D2} &= I_{DB} + I_{DU} \\ I_{C3} &= I_{CU} - I_{CB} \qquad & I_{D4} &= I_{DU} - I_{DB} \end{aligned} \tag{2-10}$$

Combining (2-1) and (2-9),

$$\begin{aligned} V_{AB} &= \tfrac{1}{2}(V_1 - V_2) = \tfrac{1}{2}(V_{CB} + V_{CU} - V_{DB} - V_{DU}) \\ V_{AU} &= \tfrac{1}{2}(V_1 + V_2) = \tfrac{1}{2}(V_{CB} + V_{CU} + V_{DB} + V_{DU}) \\ V_{BB} &= \tfrac{1}{2}(V_3 - V_4) = \tfrac{1}{2}(V_{CU} - V_{CB} - V_{DU} + V_{DB}) \end{aligned} \tag{2-11}$$

$$V_{BU} = \frac{1}{2}(V_3 + V_4) = \frac{1}{2}(V_{CU} - V_{CB} + V_{DU} - V_{DB})$$

And combining (2-8) and (2-10),

$$\begin{aligned}
V_{AB}Y_{AB} &= \frac{1}{2}(I_{DB} + I_{DU} - I_{CB} - I_{CU}) \\
V_{AU}Y_{AU} &= \frac{1}{2}(I_{EU} - I_{CB} - I_{CU} - I_{DB} - I_{DU}) \qquad (2\text{-}12) \\
V_{BB}Y_{BB} &= \frac{1}{2}(I_{DU} - I_{DB} - I_{CU} + I_{CB}) \\
V_{BU}Y_{BU} &= \frac{1}{2}(I_{EU} - I_{CU} + I_{CB} - I_{DU} + I_{DB})
\end{aligned}$$

Then using (2-5) in (2-11) and (2-12) and combining

$$\begin{aligned}
&V_{CB}(Y_{AB}+Y_{CB}) + V_{CU}(Y_{AB}+Y_{CU}) - V_{DB}(Y_{AB}+Y_{DB}) - V_{DU}(Y_{AB}+Y_{DU}) = 0 \\
&V_{CB}(Y_{AU}+Y_{CB}) + V_{CU}(Y_{AU}+Y_{CU}) + V_{DB}(Y_{AU}+Y_{DB}) + V_{DU}(Y_{AU}+Y_{DU}) = I_{EU} \\
&-V_{CB}(Y_{BB}+Y_{CB}) + V_{CU}(Y_{BB}+Y_{CU}) + V_{DB}(Y_{BB}+Y_{DB}) - V_{DU}(Y_{BB}+Y_{DU}) = 0 \\
&-V_{CB}(Y_{BU}+Y_{CB}) + V_{CU}(Y_{BU}+Y_{CU}) - V_{DB}(Y_{BU}+Y_{DB}) + V_{DU}(Y_{BU}+Y_{DU}) = I_{EU}
\end{aligned} \qquad (2\text{-}13)$$

Equations (2-13) represent four equations relating the four voltages, and may be solved simultaneously by the determinant method. The determinant of the set may be evaluated by expansion in terms of its minors, and becomes:

$$\begin{aligned}
\Delta = {} & 2(Y_{AB} - Y_{BB})(Y_{AU} - Y_{BU})(Y_{CB} - Y_{DB})(Y_{CU} - Y_{DU}) \\
& - (U + DB,DU)(B + CB,CU) - (U + CU,DB)(B + CB,DU) \\
& - (U + CB,DU)(B + CU,DB) - (U + CB,CU)(B + DB,DU)
\end{aligned} \qquad (2\text{-}14)$$

where the factors in parentheses are defined as follows

$$(B + P,Q) = (Y_{AB} + Y_P)(Y_{BB} + Y_Q) + (Y_{AB} + Y_Q)(Y_{BB} + Y_P)$$

$$= 2Y_{AB}Y_{BB} + 2Y_PY_Q + (Y_{AB} + Y_{BB})(Y_P + Y_Q) \tag{2-15}$$

$$(U + P,Q) = (Y_{AU} + Y_P)(Y_{BU} + Y_Q) + (Y_{AU} + Y_Q)(Y_{BU} + Y_P)$$

$$= 2Y_{AU}Y_{BU} + 2Y_PY_Q + (Y_{AU} + Y_{BU})(Y_P + Y_Q)$$

The solution for the four detector voltages becomes:

$$V_{CB} = \frac{I_{EU}}{\Delta}\left[\begin{array}{l}(Y_{AU} - Y_{BU})\left\{(B + DB,CU) + (B + DB,DU)\right\} \\ - (Y_{AB} - Y_{BB})(Y_{CU} - Y_{DU})(Y_{AU} + Y_{BU} + 2Y_{DB})\end{array}\right] \tag{2-16a}$$

$$V_{CU} = \frac{I_{EU}}{\Delta}\left[\begin{array}{l}(Y_{AB} - Y_{BB})(Y_{AU} - Y_{BU})(Y_{CB} - Y_{DB}) - (B + DB,DU) \times \\ (Y_{AU} + Y_{BU} + 2Y_{CB}) - (B + CB,DU)(Y_{AU} + Y_{BU} + 2Y_{DB})\end{array}\right] \tag{2-16b}$$

$$V_{DB} = \frac{I_{EU}}{\Delta}\left[\begin{array}{l}(Y_{AU} - Y_{BU})\left\{(B + CB,CU) + (B + CB,DU)\right\} \\ + (Y_{AB} - Y_{BB})(Y_{CU} - Y_{DU})(Y_{AU} + Y_{BU} + 2Y_{CB})\end{array}\right] \tag{2-16c}$$

$$V_{DU} = \frac{-I_{EU}}{\Delta}\left[\begin{array}{l}(Y_{AB} - Y_{BB})(Y_{AU} - Y_{BU})(Y_{CB} - Y_{DB}) + (B + CU,DB) \times \\ (Y_{AU} + Y_{BU} + 2Y_{CB}) + (B + CU,CB)(Y_{AU} + Y_{BU} + 2Y_{DB})\end{array}\right] \tag{2-16d}$$

2. For the case with $I_{EU} = 0$, from (2-4)

$$I_{E1} = - I_{E2} = I_{E3} = - I_{E4} = \frac{I_{EB}}{2} \tag{2-17}$$

and from (2-3)

$$I_{A1} + I_{C1} = - I_{A2} - I_{D2} = I_{B3} + I_{C3} = -I_{B4} - I_{D4} = \frac{1}{2} I_{EB} \tag{2-18}$$

I_{An} and I_{Bn} are then eliminated from (2-2) to give:

$$I_{AB} = Y_{AB}V_{AB} = \frac{1}{2}(I_{EB} - I_{C1} + I_{D2})$$
$$I_{AU} = Y_{AU}V_{AU} = -\frac{1}{2}(I_{C1} + I_{D2}) \qquad (2\text{-}19)$$
$$I_{BB} = Y_{BB}V_{BB} = \frac{1}{2}(I_{EB} - I_{C3} - I_{D4})$$
$$I_{BU} = Y_{BU}V_{BU} = -\frac{1}{2}(I_{C3} + I_{D4})$$

Using (2-9), (2-10), (2-11), and (2-5) to eliminate the desired quantities gives another set of four equations relating the detector voltages, as follows:

$$V_{CB}(Y_{AB} + Y_{CB}) + V_{CU}(Y_{AB} + Y_{CU}) - V_{DB}(Y_{AB} + Y_{DB}) - V_{DU}(Y_{AB} + Y_{DU}) = I_{EB}$$
$$V_{CB}(Y_{AU} + Y_{CB}) + V_{CU}(Y_{AU} + Y_{CU}) + V_{DB}(Y_{AU} + Y_{DB}) + V_{DU}(Y_{AU} + Y_{DU}) = 0$$
$$-V_{CB}(Y_{BB} + Y_{CB}) + V_{CU}(Y_{BB} + Y_{CU}) + V_{DB}(Y_{BB} + Y_{DB}) - V_{DU}(Y_{BB} + Y_{DU}) = I_{EB}$$
$$-V_{CB}(Y_{BU} + Y_{CB}) + V_{CU}(Y_{BU} + Y_{CU}) - V_{DB}(Y_{BU} + Y_{DB}) + V_{DU}(Y_{BU} + Y_{DU}) = 0$$

(2-20)

The determinant of these equations is the same as for the first set, given by (2-14), and the solution becomes:

$$V_{CB} = \frac{I_{EB}}{\Delta}\left[\begin{array}{l}(Y_{AB} - Y_{BB})\{(U + DB,CU) + (U + DB,DU)\} \\ -(Y_{AU} - Y_{BU})(Y_{CU} - Y_{DU})(Y_{AB} + Y_{BB} + 2Y_{DB})\end{array}\right] \qquad (2\text{-}21a)$$

$$V_{CU} = \frac{I_{EB}}{\Delta}\left[\begin{array}{l}(Y_{AB} - Y_{BB})(Y_{AU} - Y_{BU})(Y_{CB} - Y_{DB}) - (U + DB,DU) \times \\ (Y_{AB} + Y_{BB} + 2Y_{CB}) - (U + CB,DU)(Y_{AB} + Y_{BB} + 2Y_{DB})\end{array}\right]$$

(2-21b)

$$V_{DB} = \frac{-I_{EB}}{\Delta}\left[\begin{array}{l}(Y_{AB} - Y_{BB})\{(U + CB,CU) + (U + CB,DU)\} \\ + (Y_{AU} - Y_{BU})(Y_{CU} - Y_{DU})(Y_{AB} + Y_{BB} + 2Y_{CB})\end{array}\right] \qquad (2\text{-}21c)$$

$$V_{DU} = \frac{I_{EB}}{\Delta}\left[\frac{(Y_{AB} - Y_{BB})(Y_{AU} - Y_{BU})(Y_{CB} - Y_{DB}) + (U + CU,DB)x}{(Y_{AB} + Y_{BB} + 2Y_{CB}) + (U + CU,CB)(Y_{AB} + Y_{BB} + 2Y_{DB})}\right] \tag{2-21d}$$

By the theory of superposition, the total voltage at each of the detectors at C and D is the sum of the components given by (2-16) and (2-21).

The behavior of the denominator can be determined by expansion of (2-14) into the following:

$$\Delta = -4\left[\begin{aligned}&4(Y_{AB}Y_{AU}Y_{BB}Y_{BU} + Y_{CB}Y_{CU}Y_{DB}Y_{DU})\\&+(Y_{AB} + Y_{AU} + Y_{BB} + Y_{BU})\left\{Y_{CB}Y_{CU}(Y_{DB} + Y_{DU}) + Y_{DB}Y_{DU}(Y_{CB} + Y_{CU})\right\}\\&+(Y_{CB} + Y_{CU} + Y_{DB} + Y_{DU})\left\{Y_{AB}Y_{AU}(Y_{BB} + Y_{BU}) + Y_{BB}Y_{BU}(Y_{AB} + Y_{AU})\right\}\\&+(Y_{AB}Y_{BB} + Y_{AU}Y_{BU})(Y_{CB}Y_{CU} + Y_{CB}Y_{DU} + Y_{DB}Y_{CU} + Y_{DB}Y_{DU})\\&+(Y_{AB}Y_{BU} + Y_{BB}Y_{AU})(Y_{CB}Y_{CU} + Y_{CB}Y_{DB} + Y_{CU}Y_{DU} + Y_{DB}Y_{DU}\\&+(Y_{AB}Y_{AU} + Y_{BB}Y_{BU})(Y_{CB}Y_{DB} + Y_{CB}Y_{DU} + Y_{DB}Y_{CU} + Y_{DB}Y_{DU})\end{aligned}\right] \tag{2-22}$$

which is seen to be a very well-behaved function with no apparent singularities.

A particular case of interest occurs when the admittances of the two detectors are identical, i.e., $Y_{CB} = Y_{DB}$ and $Y_{CU} = Y_{DU}$. For this case, the detector voltages are:

$$V_{CB} = \frac{2I_{EU}}{\Delta}(Y_{AU} - Y_{BU})(B + CB,CU) + \frac{2I_{EB}}{\Delta}(Y_{AB} - Y_{BB})(U + CB,CU)$$

$$= \frac{1}{2}\left[I_{EU}\frac{(Y_{AU} - Y_{BU})}{(U + CB,CU)} + I_{EB}\frac{(Y_{AB} - Y_{BB})}{(B + CB,CU)}\right] \tag{2-23a}$$

$$V_{CU} = \frac{-2I_{EU}}{\Delta}(Y_{AU}+Y_{BU}+2Y_{CB})(B+CB,CU) - \frac{-2I_{EB}}{\Delta}(Y_{AB}+Y_{BB}+2Y_{CB})(U+CB,CU)$$

$$= \frac{1}{2}\left[-I_{EU}\frac{(Y_{AU}+Y_{BU}+2Y_{CB})}{(U+CB,CU)} - I_{EB}\frac{(Y_{AB}+Y_{BB}+2Y_{CB})}{(B+CB,CU)}\right] \qquad (2\text{-}23b)$$

$$V_{DB} = \frac{2I_{EU}}{\Delta}(Y_{AU}-Y_{BU})(B+CB,CU) - \frac{2I_{EB}}{\Delta}(Y_{AB}-Y_{BB})(U+CB,CU)$$

$$= \frac{1}{2}\left[I_{EU}\frac{(Y_{AU}-Y_{BU})}{(U+CB,CU)} - I_{EB}\frac{(Y_{AB}-Y_{BB})}{(B+CB,CU)}\right] \qquad (2\text{-}23c)$$

$$V_{DU} = \frac{-2I_{EU}}{\Delta}(Y_{AU}+Y_{BU}+2Y_{CB})(B+CB,CU) + \frac{2I_{EB}}{\Delta}(Y_{AB}+Y_{BB}+2Y_{CB})(U+CB,CU)$$

$$= \frac{1}{2}\left[-I_{EU}\frac{(Y_{AU}+Y_{BU}+2Y_{CB})}{(U+CB,CU)} + I_{EB}\frac{(Y_{AB}+Y_{BB}+2Y_{CB})}{(B+CB,CU)}\right] \qquad (2\text{-}23d)$$

An examination of equations (2-23) leads to the following conclusions concerning operation of the shielded two-wire hybrid junction as an impedance bridge:

(1) The balanced components of detector voltage, V_{CB} and V_{DB}, do exhibit nulls when the load admittances Y_A and Y_B are identical. Furthermore, for balanced input currents I_{EB}, this null is dependent only on the identity of the balanced components of load admittances, Y_{AB} and Y_{BB}; and for unbalanced input currents, the null is dependent only on identity of unbalanced load components, Y_{AU} and Y_{BU}. If interest is centered chiefly in measurement of balanced components of admittance, an effort should be made to provide only balanced input currents. This is particularly true if the adjustable standard admittance does not permit independent adjustment of its balanced and unbalanced components. It should also be noticed that the phase of the balanced detector voltages is the same for contributions from unbalanced input currents, but opposite for balanced input currents; this fact may be of use in differentiating against the effects of unbalanced input currents.

(2) It is apparent that the unbalanced components of detector voltage do not exhibit a null for identical load impedances, and in fact would disappear only for a very special combination of both balanced and unbalanced inputs. Consequently, if the above null property is to be made use of, detectors must be used which respond only to balanced voltages, and differentiate very sharply against unbalanced components in order to indicate a null of the balanced components.

(3) Finally, the conditions imposed on the solution (2-23) should be reviewed. These are that the detector admittances be identical, including both balanced and unbalanced components. This places certain restrictions on the detectors themselves, as well as any combining network incorporated to take advantage of the relative phase of the voltage components. In general, it would also be advantageous to terminate both modes at the detectors in something approaching their characteristic impedances, in order to reduce reflections and standing waves in the detector arms of the hybrid junction.

The only other outstanding assumption involved is that the loads do not couple modes; even if this is not true, qualitative aspects of the performance of the hybrid junction may still be obtained from Eqs. (2-23).

Bibliography

1. Tomiyasu, K., "Unbalanced Terminations on a Shielded-Pair Line," Cruft Laboratory, Technical Report No. 86 (1949).

2. Smythe, W.R., "Static and Dynamic Electricity," McGraw-Hill, New York, 512 (1950).

3. Frankel, S., "Parallel Wires in Rectangular Troughs," Proc. I. R. E. 30, 182-190 (1942).

4. Southwell, R. V., Relaxation Methods for Engineering Sciences, Clarendon Press, London (1940).

5. King, R. W. P., Electromagnetic Engineering, Vol. I, McGraw-Hill, New York (1945) p. 348.

6. Cutrona, L. J., "The Theory of Biconjugate Networks," Proc. I. R. E. 39, 827-832 (1951).

7. Tyrell, W. A., "Hybrid Circuits for Microwaves," Proc. I. R. E. 35, 1294-1306 (1947).

8. Morita, T., Sheingold, S., "A Coaxial Magic-T," Cruft Laboratory Technical Report No. 162 (1952).

DISTRIBUTION LIST

Technical Reports

2	Chief of Naval Research (427) Department of the Navy Washington 25, D. C.
1	Chief of Naval Research(460) Department of the Navy Washington 25, D. C.
1	Chief of Naval Research (421) Department of the Navy Washington 25, D. C.
6	Director (Code 2000) Naval Research Laboratory Washington 25, D. C.
2	Commanding Officer Office of Naval Research Branch Office 150 Causeway Street Boston, Massachusetts
1	Commanding Officer Office of Naval Research Branch Office 1000 Geary Street San Francisco 9, California
1	Commanding Officer Office of Naval Research Branch Office 1030 E. Green Street Pasadena, California
1	Commanding Officer Office of Naval Research Branch Office The John Crerar Library Building 86 East Randolph Street Chicago 1, Illinois
1	Commanding Officer Office of Naval Research Branch Office 346 Broadway New York 13, New York
3	Officer-in-Charge Office of Naval Research Navy No. 100 Fleet Post Office New York, N. Y.

1	Chief, Bureau of Ordnance (Re4) Navy Department Washington 25, D. C.
1	Chief, Bureau of Ordnance (AD-3) Navy Department Washington 25, D. C.
1	Chief, Bureau of Aeronautics (EL-1) Navy Department Washington 25, D. C.
2	Chief, Bureau of Ships (810) Navy Department Washington 25, D. C.
1	Chief of Naval Operations (Op-413) Navy Department Washington 25, D. C.
1	Chief of Naval Operations (Op-20) Navy Department Washington 25, D. C.
1	Chief of Naval Operations (Op-32) Navy Department Washington 25, D. C.
1	Director Naval Ordnance Laboratory White Oak, Maryland
2	Commander U. S. Naval Electronics Laboratory San Diego, California
1	Commander (AAEL) Naval Air Development Center Johnsville, Pennsylvania
1	Librarian U. S. Naval Post Graduate School Monterey, California
50	Director Signal Corps Engineering Laboratories Evans Signal Laboratory Supply Receiving Section Building No. 42 Belmar, New Jersey

3	Commanding General (RDRRP) Air Research and Development Command Post Office Box 1395 Baltimore 3, Maryland
2	Commanding General (RDDDE) Air Research and Development Command Post Office Box 1395 Baltimore 3, Maryland
1	Commanding General (WCRR) Wright Air Development Center Wright-Patterson Air Force Base, Ohio
1	Commanding General (WCRRH) Wright Air Development Center Wright-Patterson Air Force Base, Ohio
1	Commanding General (WCRE) Wright Air Development Center Wright-Patterson Air Force Base, Ohio
2	Commanding General (WCRET) Wright Air Development Center Wright-Patterson Air Force Base, Ohio
1	Commanding General (WCREO) Wright Air Development Center Wright-Patterson Air Force Base, Ohio
2	Commanding General (WCLR) Wright Air Development Center Wright-Patterson Air Force Base, Ohio
1	Commanding General (WCLRR) Wright Air Development Center Wright-Patterson Air Force Base, Ohio
2	Technical Library Commanding General Wright Air Development Center Wright-Patterson Air Force Base, Ohio
1	Commanding General (RCREC-4C) Rome Air Development Center Griffiss Air Force Base Rome, New York
1	Commanding General (RCR) Rome Air Development Center Griffiss Air Force Base Rome, New York

2 Commanding General (RCRW)
Rome Air Development Center
Griffiss Air Force Base
Rome, New York

6 Commanding General (CRR)
Air Force Cambridge Research Center
230 Albany Street
Cambridge 39, Massachusetts

1 Commanding General
Technical Library
Air Force Cambridge Research Center
230 Albany Street
Cambridge 39, Massachusetts

2 Director
Air University Library
Maxwell Air Force Base, Alabama

1 Commander
Patrick Air Force Base
Cocoa, Florida

2 Chief, Western Division
Air Research and Development Command
P. O. Box 2035
Pasadena, California

1 Chief, European Office
Air Research and Development Command
Shell Building
60 Rue Ravenstein
Brussels, Belgium

1 U. S. Coast Guard (EEE)
1300 E Street, N. W.
Washington, D. C.

1 Assistant Secretary of Defense
(Research and Development)
Research and Development Board
Department of Defense
Washington 25, D. C.

5 Armed Services Technical Information Agency
Document Service Center
Knott Building
Dayton 2, Ohio

1 Director
Division 14, Librarian
National Bureau of Standards
Connecticut Avenue and Van Ness St., N. W.

1 Director
Division 14, Librarian
National Bureau of Standards
Connecticut Avenue and Van Ness St., N. W.

1 Office of Technical Services
Department of Commerce
Washington 25, D. C.

1 Commanding Officer and Director
U. S. Underwater Sound Laboratory
New London, Connecticut

1 Federal Telecommunications Laboratories, Inc.
Technical Library
500 Washington Avenue
Nutley, New Jersey

1 Librarian
Radio Corporation of America
RCA Laboratories
Princeton, New Jersey

1 Sperry Gyroscope Company
Engineering Librarian
Great Neck, L. I., New York

1 Watson Laboratories
Library
Red Bank, New Jersey

1 Professor E. Weber
Polytechnic Institute of Brooklyn
99 Livingston Street
Brooklyn 2, New York

1 University of California
Department of Electrical Engineering
Berkeley, California

1 Dr. E. T. Booth
Hudson Laboratories
145 Palisade Street
Dobbs Ferry, New York

1 Cornell University
Department of Electrical Engineering
Ithaca, New York

1 University of Illinois
Department of Electrical Engineering
Urbana, Illinois

1 Johns Hopkins University
Applied Physics Laboratory
Silver Spring, Maryland

1 Professor A. von Hippel
Massachusetts Institute of Technology
Research Laboratory for Insulation Research
Cambridge, Massachusetts

1 Director
Lincoln Laboratory
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

1 Signal Corps Liaison Office
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

1 Mr. Hewitt
Massachusetts Institute of Technology
Document Room
Research Laboratory of Electronics
Cambridge, Massachusetts

1 Stanford University
Electronics Research Laboratory
Stanford, California

1 Professor A. W. Straiton
University of Texas
Department of Electrical Engineering
Austin 12, Texas

1 Yale University
Department of Electrical Engineering
New Haven, Connecticut

1 Mr. James F. Trosch, Administrative Aide
Columbia Radiation Laboratory
Columbia University
538 West 120th Street
New York 27, N. Y.

1 Dr. J.V.N. Granger
Stanford Research Institute
Stanford, California